Statistical Crack Propagation in Fastener Holes Under Spectrum Loading

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A simple crack growth rate-based statistical model for fatigue crack propagation in fastener holes under spectrum loadings is proposed and investigated. A method for analyzing the crack growth rate data in order to calibrate the model parameters is presented. Emphasis is placed upon crack propagation in the small crack size region where the statistical dispersion is very significant. The statistical distributions of both the crack size at any service time instant and the crack propagation life to reach any specific crack size are derived analytically. Available fractographic data for 7475-T7351 aluminum fastener hole specimens under B-1 bomber loading spectra have been analyzed statistically. It is demonstrated that the correlation between the statistical model and the fractographic results is very reasonable.

Introduction

FATIGUE cracking in fastener holes represents one of the most important problems in the design of airframe structures, such as lower wing skins. Under well controlled laboratory test conditions, experimental test results for coupon specimens and full-scale articles indicate a significant statistical variability in fatigue crack growth damage accumulation. Two sets of actual crack propagation records in the small crack size region for 7475-T7351 aluminum fastener hole specimens[1] under a bomber loading spectrum are shown in Figs. 1a and 1b, referred to as the WPB and XWPB data sets, respectively, as will be explained later. Therefore, the application of statistical approaches to fatigue crack propagation recently has received considerable attention [i.e., Refs. 2-9].

The objective of this paper is to investigate the statistics of crack propagation in fastener holes which were subjected to service loading spectra. In particular, attention is focused on crack growth damage accumulation in the small crack size region for the purpose of durability analyses.

A crack growth rate-based statistical model, for fatigue crack propagation in fastener holes under spectrum loading, is proposed and investigated. Based on the model, a method is presented for analyzing the available crack growth rate data in order to calibrate the model statistics and fracture mechanics parameters. Emphasis is placed upon the simplicity of the statistical model and on its suitability for direct application to practical problems, such as in a durability analysis. Although the simple statistical model developed herein is useful to other crack propagation problems, it is limited to crack growth damage accumulation in the small crack size region where the statistical dispersion is significant. For the crack propagation of aluminum specimens in a large crack size region, the present model appears to predict larger statistical dispersion and, hence, a more complex statistical model is needed, as proposed in Ref. 9. Fractographic data obtained from laboratory tests[1] have been analyzed statistically and correlated with the theoretical model.

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Based on the proposed model, the statistical distributions of the crack growth rate, the crack propagation life to reach any crack size, and the crack size at any service life, have been derived analytically. A correlation study is carried out to compare the results of the statistical model with the fractographic data. It is demonstrated that the correlation is very reasonable.

Technical Approach

For aluminum fastener hole specimens, subject to spectrum loadings such as bomber or fighter spectra, extensive fractographic results indicate that in the small crack size region, the crack growth rate equation[10-16] can be expressed as

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = Qa^b(t) \tag{1}$$

where $a(t)$ is the crack size at t flight hours, and Q and b are constants.

As mentioned previously, even under well controlled laboratory conditions, crack growth rate data exhibit considerable statistical variability. In order to account for such a statistical scatter in the prediction of crack growth damage accumulation in fastener holes, we proposed to randomize Eq. (1) as follows:

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = X(t)Qa^b(t) \tag{2}$$

where $X(t)$ is a positive random process (or stochastic process) taking values around unity. Thus, the deterministic fracture mechanics model, Eq. (1), represents the central tendency of the crack growth behavior, whereas the statistical variability is taken care of by the random process $X(t)$.

Taking the logarithm of both sides of Eq. (2), one obtains

$$Y = bU + q + Z \tag{3}$$

where

$$Y = \log\frac{\mathrm{d}a(t)}{\mathrm{d}t}, \quad U = \log a(t)$$
$$q = \log Q, \quad Z = \log X(t) \tag{4}$$

The log crack growth rate, $Y = \log[\mathrm{d}a(t)/\mathrm{d}t]$, vs the log crack size, $U = \log a(t)$, for the test results shown in Figs. 1a and 1b are presented by dots in Figs. 2a and 2b, respectively. The test results shown in Fig. 2 scatter around a straight line, indicating the validity of Eqs. (2) and (3).

Two extreme cases of the random process $X(t)$ should be mentioned. At one extreme $X(t)$ is completely independent at any two time instants, referred to as a white noise process. Based on the central limit theorem, it can be shown that the statistical variability of the crack size or the fatigue life, after integrating Eq. (2), is the smallest within the class of random processes. Hence, it is very unconservative for engineering analyses and applications, as evidenced by a Monte Carlo simulation study carried out in Ref. 4. In addition, the mathematical solution for the white noise process model is difficult. At another extreme, the random process $X(t)$ is totally correlated at any two time instants, indicating that $X(t)$ is a random variable, i.e., $X(t)=X$. For the case of random variable X, the statistical dispersion of fatigue crack growth damage accumulation is the largest in the class of random processes. Consequently, the random variable model is conservative for the prediction of crack propagation life. When $X(t)$ is a general random process, the statistical analysis for crack growth damage accumulation is rather involved.[8,9]

Because of mathematical simplicity for analyses and applications, and due to its conservative nature, $X(t)$ is modeled as a positive random variable following the lognormal distribution which takes values around unity. It follows, then, from Eq. (4), that $Z=\log X$ is a normal random variable with zero mean and standard deviation, σ_z. From Eq. (3) the log crack growth rate $Y=\log[da(t)/dt]$ is a normal random variable with mean value μ_y and standard deviation σ_y given by

$$\mu_y = bU + q \tag{5}$$

$$\sigma_y = \sigma_z \tag{6}$$

The parameters b and Q, as well as the standard deviation/ σ_z, of Z, conditional on the crack size a, can be estimated from the test results of the crack growth rate vs the crack size using Eq. (3) and the linear regression analysis. With the crack growth rate data shown as dots in Fig. 2, the method of linear regression is employed to estimate b, Q, and σ_z. The results aer presented in Table 1. Also displayed in Fig. 2 as straight lines are the mean values of the log crack growth rate, μ_y, given by Eq. (5). Since Y and Z are normal random variables, and Eq. (3) is linear, the linear regression analysis is identical to the method of least squares.

The crack growth rate data shown in Fig. 2 were obtained from the crack length-flight hours test results using the modified secant method that was recommended in Ref. 4. Indeed, the variability in crack growth rates depends on the growth rate data reduction procedures, such as direct secant method, seven points polynomial, etc. The statistical significance and the variability in crack growth rate with respect to various growth rate data reduction procedures will be reported in a future paper.

To show the validity of the assumption that Z follows the normal distribution, sample values of Z, denoted by z_j, are computed from the sample values of Y and U, denoted by (y_j, u_j), using Eq. (3),

$$z_j = y_j - bu_j - q \quad \text{for } j = 1,2,\ldots,n \tag{7}$$

where b and q have bee estimated by the linear regression analysis and n is the total number of test data.

Sample data, z_j $(j=1,2,\ldots,n)$, associated with Fig. 2 are computed from Eq. (7) and plotted on normal probability paper in Fig. 3, where the sample values, z_j, are arranged in ascending order, viz, $z_1, \leq z_2 \leq \ldots \leq z_n$. The ordinate corresponding to z_j is given by $\phi^{-1}[j/(n+1)]$ with $\phi^{-1}(\ )$ being the inversed standardized normal distribution function. A straight line shown in Fig. 3 denotes the normal distribution for Z with σ_z given in Table 1. It is observed that the sample values of Z scatter around the straight line, indicating that the normal distribution is reasonable.

Kolmogorov-Smirnov tests for goodness of fit have been performed to determine the observed K-S statistics. These results show that the normal distribution is acceptable at least as a 20% level of significance for both the WPB and XWPB data sets, indicating an excellent fit for normal distribution.

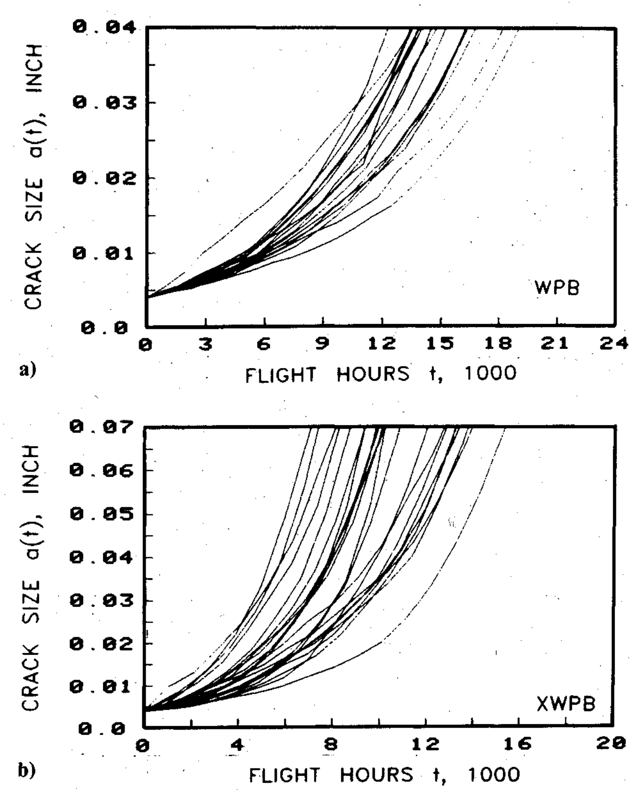

Fig. 1 Actual crack propagation time histories of fastener hole specimens: a) WPB data set and b) XWPB data set.

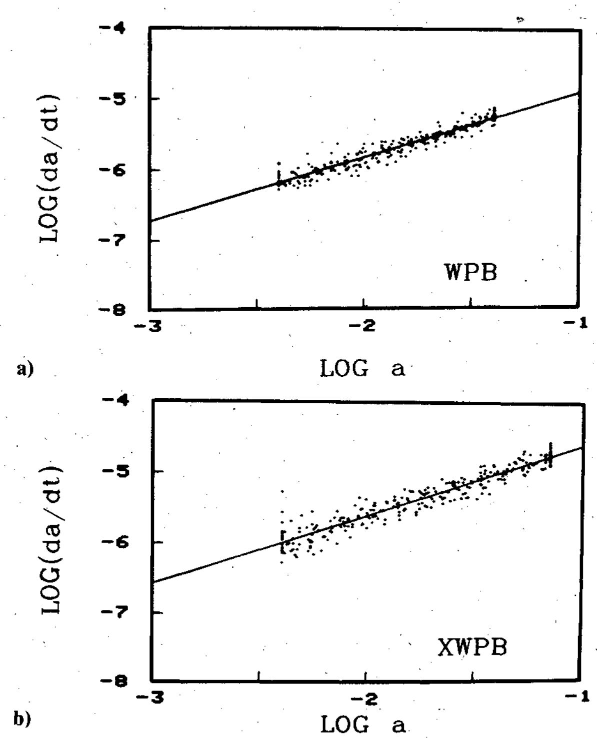

Fig. 2 Crack growth rate as function of crack size: a) WPB data set and b) XWPB data set.

Table 1 **Linear Regression Estimate of b, Q, σ_z and coefficient of variation V of crack growth rate for the WPB and XWPB data sets**

Data set	b	Q	σ_z	V,%	No. of data,n
WPB	0.9297	1.105×10^{-4}	0.0876	20.38	264
XWPB	0.9850	$2.44\ \times10^{-4}$	0.1290	30.37	273

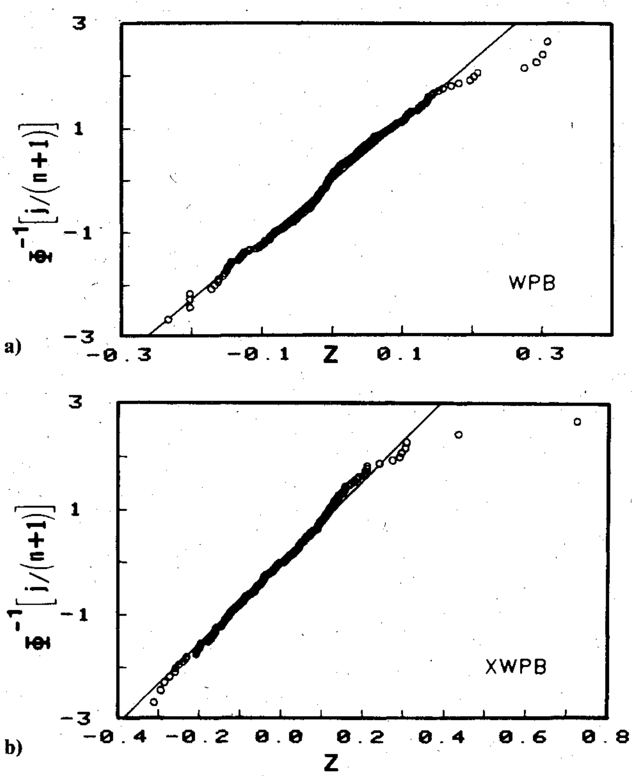

Fig. 3 Normal probability plots for Z: a) WPB data set and b) XWPB data set.

Since $Z=\log X$ and $Y=\log[\mathrm{d}a(t)/\mathrm{d}t]$ are normal random variables, the crack growth rate, $G=\mathrm{d}a(t)/\mathrm{d}t$, follows a lognormal distribution. The coefficient of variation V, of $G=\mathrm{d}a(t)/\mathrm{d}t$, is related to the standard deviation, $\sigma_y=\sigma_z$, through

$$V=\left[e^{(\sigma_z \ln 10)^2}-1\right]^{1/2} \tag{8}$$

The coefficients of variation V of the crack growth rate for the WPB and XWPB data sets are shown in Table 1.

Equation (2) can be integrated to yield the crack size $a(t)$ as a function of flight hours t,

$$a(t)=a(0)/[1-XcQta^c(0)]^{1/c} \tag{9}$$

in which $a(0)$ is the initial crack size and

$$c=b-1 \tag{10}$$

Let z_γ be the γ percentile of Z, i.e.,

$$\gamma\%=P[Z>z_\gamma]=1-\Phi(z_\gamma/\sigma_z)$$

or, conversely,

$$z_\gamma=\sigma_z\Phi^{-1}(1-\gamma\%) \tag{11}$$

Then, the γ percentile of the random variable X, denoted by x_γ, is given by

$$x_\gamma=(10)^{z_\gamma} \tag{12}$$

and the γ percentile of the crack size, $a_\gamma(t)$, at t flight hours follows from Eqs. (9) and (12) as

$$a_\gamma(t)=a(0)/[1-x_\gamma cQta^c(0)]^{1/c} \tag{13}$$

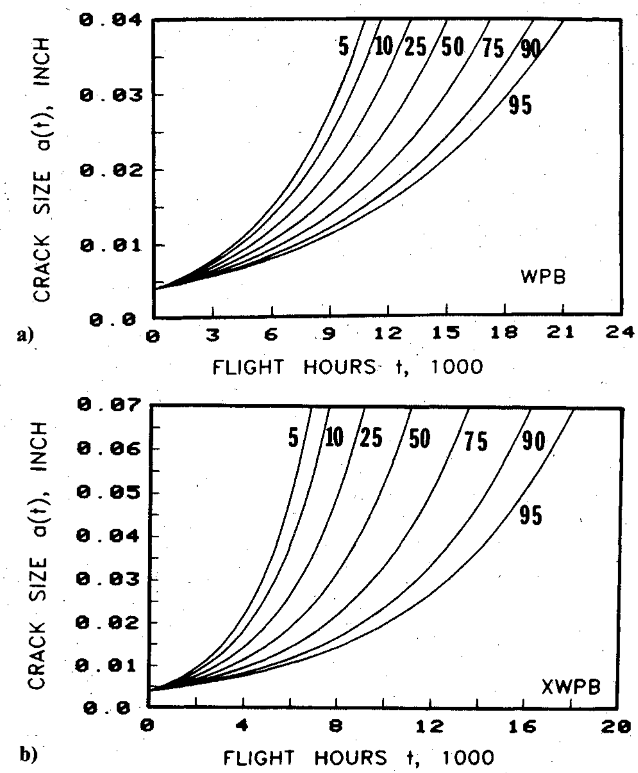

Fig. 4 Percentiles of crack size $a(t)$ as function of service time t (theoretical model) for a) WPB data set and b) XWPB data set.

Various γ percentiles of the crack size, $a_\gamma(t)$, vs flight hours t, have been computed from Eqs. (10-13), using $a(0)=0.1$ mm (0.004 in.) and Table 1 values for the WPB and XWPB data sets. The results are displayed in Fig. 4. By way of example, the curves associated with $\gamma=10$, in Fig. 4, indicate that the probability is 10% that a specimen will have a crack size growing faster than that shown by the curve.

The distribution function of X is lognormal and is given by

$$F_X(x)=P[X\le x]=\Phi\left(\frac{\log x}{\sigma_z}\right) \tag{14}$$

where $\Phi(\ )$ is the standardized normal distribution function and σ_z has been estimted previously.

In the prediction of crack growth damage accumulation in fastener holes, two statistical distributions are most important: the distribution of the crack size at any service time t, and the distribution of service life to reach any given crack size including the critical crack size. Based on the theoretical model developed herein, these two distributions can be derived analytically in the following manner.

The distribution function of the crack size, $a(t)$, at any specified number of flight hours, t, can be obtained from that of X given by Eq. (14), and using the transformation of Eq. (9). This results in the distribution

$$F_{a(t)}(x)=P[a(t)\le x]=\Phi\left[\log\left(\frac{a^{-c}(0)-x^{-c}}{cQt}\right)\Big/\sigma_z\right] \tag{15}$$

The probability that the crack size will exceed any specific value x, denoted by $F^*_{a(t)}(x)$, is referred to as the crack exceedance curve,

$$F^*_{a(t)}(x),=P[a(t)>x]=1-F_{a(t)}(x) \tag{16}$$

in which $F_{a(t)}(x)$ is given by Eq. (15).

Let $T(a_1)$ be the time to reach any given crack size, a_1. Obviously, $T(a_1)$ is a statistical variable and can be obtained from Eq. (9) by replacing t and $a(t)$ by $T(a_1)$ and a_1,

Table 2 Comparison between experimental test results and predicted variabilities in propagation lives (flight hours) to reach given crack sizes

Percentiles	WPB data set crack size = 0.02 in. Test data, 10^4	Statistical model, 10^4	Percentiles	XWPB data set crack size = 0.025 in. Test data, 10^4	Statistical model, 10^4
0.058	0.736	0.751	0.043	0.418	0.421
0.117	0.863	0.811	0.087	0.422	0.468
0.176	0.865	0.855	0.130	0.445	0.501
0.235	0.913	0.891	0.174	0.462	0.530
0.294	0.915	0.925	0.217	0.546	0.555
0.353	0.937	0.956	0.261	0.584	0.579
0.411	0.983	0.986	0.304	0.607	0.602
0.470	1.007	1.016	0.348	0.616	0.624
0.529	1.011	1.047	0.391	0.634	0.645
0.588	1.064	1.079	0.435	0.636	0.667
0.647	1.109	1.113	0.478	0.640	0.689
0.705	1.132	1.150	0.522	0.691	0.712
0.764	1.140	1.192	0.565	0.747	0.735
0.823	1.153	1.243	0.609	0.753	0.761
0.882	1.253	1.310	0.652	0.779	0.787
0.941	1.394	1.414	0.696	0.813	0.816
			0.739	0.816	0.847
			0.783	0.822	0.884
			0.826	0.842	0.926
			0.869	0.857	0.978
			0.913	0.883	1.049
			0.956	1.103	1.163

respectively, i.e.,

$$T(a_I) = \frac{1}{cQX}[a^{-c}(0) - a_I^{-c}] \tag{17}$$

Thus, the distribution function of $T(a_I)$ can be obtained from that of X given by Eq. (14) through the transformation of Eq. (17); resulting in

$$F_{T(a_I)}(\tau) = P[T(a_I) \le \tau] = 1 - \Phi\left(\frac{\log\eta}{\sigma_z}\right) \tag{18}$$

where

$$\eta = \frac{1}{cQ\tau}[a^{-c}(0) - a_I^{-c}] \tag{19}$$

The theoretical distributions for the crack size at any given number of flight hours and the time to reach any specific crack size derived above require only the crack growth rate data (shown in Fig. 2) for determining the fracture mechanics parameters b and Q, and the model statistics $\sigma_y = \sigma_z$ (Table 1).

Correlation with Test Results

The fractographic data for the 7475-T7351 aluminum fastener hole specimens under spectrum loading, and which provide the data base for the statistical analysis presented herein, are from Ref. 1. One of the data groups from that reference, referred to as the WPB data set, indicates that the fastener hole specimens from that group do not demonstrate load transfer characteristics. The holes were drilled using Winslow Spacemetric (W) machines with proper (P) drilling technique and subject to B-1 bomber spectra (B). Another group of data, referred to as the XWPB data set, has the same meaning except that the amount of load transfer is 15% (X). Both of the fractographic data sets presented in Ref. 1 were censored to include only those specimens having fatigue crack growth through the crack length intervals from 0.10 to 1.02 mm (0.004 to 0.04 in.) for the WPB data set and from 0.10 to 1.78 mm (0.004 to 0.07 in.) for the XWPB data set. This censoring procedure is necessary in order to normalize the data sets to zero life at 0.10 mm (0.004 in.) so as to obtain the homogeneous data bases shown in Fig. 1a and 1b.

Based on the statistical model, the distributions of the crack growth damage accumulation, $a(t)$, as a function of flight hours, t, can be established using Eqs. (10-13) and expressed n terms of γ percentiles by varying the value of γ as described

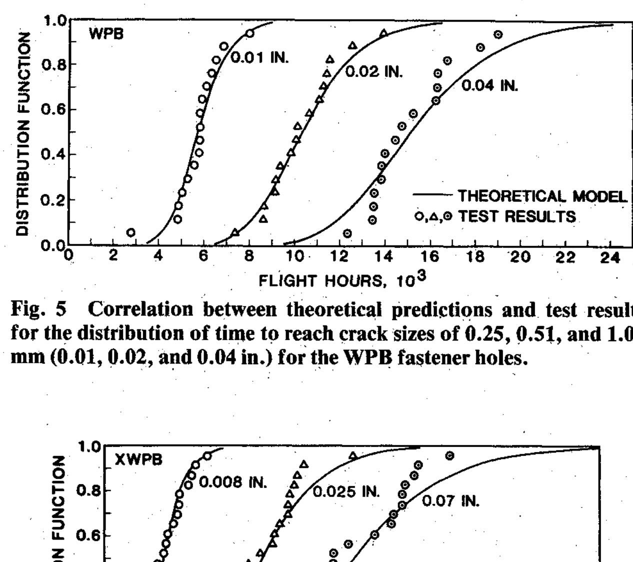

Fig. 5 Correlation between theoretical predictions and test results for the distribution of time to reach crack sizes of 0.25, 0.51, and 1.02 mm (0.01, 0.02, and 0.04 in.) for the WPB fastener holes.

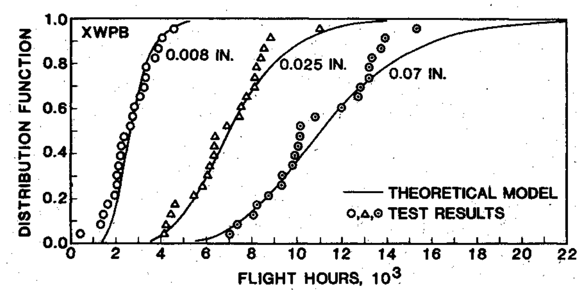

Fig. 6 Correlation between theoretical predictions and test results for the distribution of time to reach crack sizes of 0.20, 0.64, and 1.78 mm (0.008, 0.025, and 0.07 in.) for the XWPB fastener holes.

previously. They are shown in Figs. 4a and 4b, respectively, for the WPB and XWPB data sets. Although a visual comparison between Figs. 4a and 1a as well as between Figs. 4b and 1b indicates a reasonable correlation, the theoretical model appears to give slightly conservative results. The variabilities in propagation lives to reach two given crack sizes (0.02 in. for WPB data set and 0.025 in. for XWPB data set) are shown in Table 2 for theoretical predictions and experimental test results.

For further correlation study the statistical distribution for the number of flight hours to reach a certain crack size, and

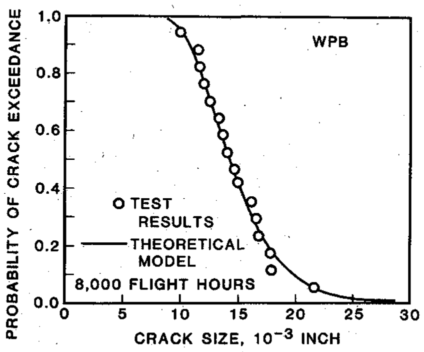

Fig. 7 Correlation between theoretical predictions and test results for the probability of crack exceedance at 8000 flight hours, WPB fastener holes.

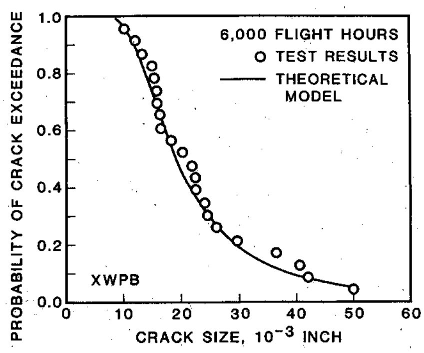

Fig. 8 Correlation between theoretical predictions and test results for the probability of crack exceedance at 6000 flight hours, XWPB fastener holes.

the crack exceedance probability at any specified number of flight hours, have been derived analytically in Eqs. (18), (19), (15), and (16). Based on the statistical model [Eq. (18)], the distributions for the number of flight hours to reach crack sizes 0.25, 0.51, and 1.02 mm (0.01, 0.02 and 0.04 in.) are plotted in Fig. 5 as solid curves for the WPB data set. Similarly the distributions for the number of flight hours to reach 0.20, 0.64, and 1.78 mm (0.008, 0.025, and 0.07 in.) cracks for the XWPB data set are displayed in Fig. 6 as solid curves. The corresponding test results obtained from Fig. 1 are shown in Figs. 5 and 6 as circles, triangles, and dotted circles. Figures 5 and 6 demonstrate that the correlation between the statistical model (solid curves) and the test results (circles, triangles, and dotted circles) is reasonable. The statistical variability based on the theoretical model appears to be slightly larger at longer crack size condition.

The crack exceedance curves based on the statistical model [Eq. (16)], for the WPB data set at 8000 flight hours, and the XWPB data set at 6000 flight hours, are plotted in Figs. 7 and 8, respectively, as solid curves. Also shown in these figures as circles are the corresponding test results obtained from Fig. 1. It is observed from Figs. 7 and 8 that the correlation between the statistical model and the test results is very reasonable.

Conclusions

A simple crack growth rate-based statistical model for fatigue crack growth damage accumulation of fastener holes subject to service loading spectra is proposed. The linear regression analysis is used to estimate the fracture mechanics parameters from a Paris-type crack growth relation and to calibrate the statistical properties of the model. The resultant statistical model is applied to fractographic data of 7475-T7351 aluminum fastener hole specimens subjected to B-1 bomber spectra. A correlation study for the distributions of both the crack size at any service time and the propagation life to reach any given crack size, demonstrates reasonable correlation between the theoretical model and the fractographic crack growth results.

The linear regression estimate for a few fracture mechanics parameters and the model statistic requires only the crack growth rate data without the need for the information of how the crack size varies as a function of propagation life. Hence the statistical model requires neither a large number of test specimens nor a homogeneous data set for an effective assessment of crack growth damage accumulation. As a result, the present model is practical in view of the limited experimental data that is normally available. In addition, the model is mathematically very simple for engineering applications.

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